Enrollment No: _______________________ Exam Seat No:_______________________

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT1 **Branch: B.Tech (All)**

Semester: 1 **Date: 09/03/2021** **Time: 03:00 To 06:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If the power of x and y both areeven then the curve is symmetrical about 01
(a) X-axis (b)Y-axis (c)about both X and Y axes (d)None

b) True/False: the two tangents to the curve $y^2 = x^3$ at the origin are real and distinct. 01

c) The infinite series $1 + r + r^2 + \cdots + r^{n-1}$ is convergent if 01
(a)$|r| < 1$ (b)$|r| > 1$ (c)$|r| \geq 1$ (d)$|r| = -1$

d) If$\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(x,y)}$ is equal to 01
(a)1 (b)-1 (c)0 (d) None of these

e) Condition for $f(x, y)$ to be maximum 01
(a)$f_x = 0 = f_y, rt < s^2, r < 0$) (b)$f_x = 0 = f_y, rt > s^2, r < 0$
(c)$f_x = 0 = f_y, rt > s^2, r > 0$ (d)$f_x = 0 = f_y, rt = s^2, r > 0$

f) If $u = y^x$, then $\frac{\partial u}{\partial x}$ is 01
(a)xy^{x-1} (b)0 (c)$y^x \log x$ (d)None of these

g) If $f(x, y) = 0$, then the $\frac{dy}{dx}$ is equal to 01
(a)$\frac{f_x}{f_y}$(b)$\frac{f_y}{f_x}$ (c)$-\frac{f_y}{f_x}$ (d)$-\frac{f_x}{f_y}$

h) The number of solutions to equation $z^2 + \bar{z} = 0$ is 01
(a)1 (b)2 (c)3 (d)4

i) The polar form of the complex number $\frac{1+i}{1-i}$ is 01
(a)$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (b)$\sin\frac{\pi}{2} + i\cos\frac{\pi}{2}$
(c)$\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ (d)$\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}$

j) $\lim_{x \to 0} \frac{\tan x}{x}$ is of the form 01

(a) $\frac{0}{0}$ (b) $\frac{\infty}{\infty}$ (c) 0^0 (d) $\infty - \infty$

k) Which of thefollowing is indeterminate form 01

(a) 0^0 (b) $0 \cdot \infty$ (c) ∞^∞ (d) All

l) If $y = x^7$ then $y_7 =$ ____. 01

(a) 7! (b) $7!\,x$ (c) 0 (d) 8!

m) If $y = \sin x \cos x$ the y_n equal to 01

(a) $\frac{1}{2}(2)^n \cos\left(\frac{n\pi}{2} + 2x\right)$ (b) $\frac{1}{2}(2)^n \sin\left(\frac{n\pi}{2} + 2x\right)$

(c) $\frac{1}{2}(2)^n \sin\left(\frac{n\pi}{2} + x\right)$ (d) None of these

n) The value of $(i)^i$ is 01

(a) $e^{-\frac{\pi}{2}}$ (b) $e^{\frac{\pi}{4}}$ (c) e^2 (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a. Evaluate: $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\log x}\right)$ 05

b. If $y = \cos x \cdot \cos 2x \cdot \cos 3x$ then find y_n. 05

c. Expand $e^{\sin x}$ as a series of ascending power of x up to x^4. 04

Q-3 Attempt all questions (14)

a. Find roots common to the equation $x^4 + 1 = 0$. 05

b. Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$ 05

c. Evaluate: $\lim_{(x,y) \to (0,1)} \frac{x - xy + 3}{x^2y + 5xy - y^3}$ 04

Q-4 Attempt all questions (14)

a. If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ show that $x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) = \frac{1}{2}\sin 2u$. 06

b. Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in power of $(x - 3)$. 05

c. Evaluate: $\lim_{x \to 0} \frac{\log x^2}{\cot x^2}$ 03

Q-5 Attempt all questions (14)

a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that 05

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

b. Trace the curve $xy^2 = 4a^2(2a - x)$. 05

c. Test for convergence $\sum_{n=2}^{\infty} \frac{1}{n \log n}$. 04

Q-6 **Attempt all questions** **(14)**

a. Find the extreme value of the $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. 07

b. If $u = \sin^{-1}(x - y)$, where $x = 3t$ and $y = 4t^3$, show that 05

$$\frac{du}{dt} = \frac{3}{\sqrt{1 - t^2}}.$$

c. Show that the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n}\right)$ is convergent. 02

Q-7 **Attempt all questions** **(14)**

a. Use Taylor's series to expand $\sin x \cos y$ in a power of $\left(x - \frac{\pi}{3}\right)$ and $\left(y - \frac{\pi}{4}\right)$ 05

b. Show that $\sum_{n=1}^{\infty}\frac{n}{2^n}$converses. 05

c. Prove that $\cos h^{-1} z = \log\left(z + \sqrt{z^2 - 1}\right)$. 04

Q-8 **Attempt all questions** **(14)**

a. If $x = r\sin\theta\ \ cos\phi, y = r\sin\theta\ sin\phi, z = r\cos\theta$, find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. 05

b. Find the maximum value of $V(x, y, z) = xyz$ subject to the constraint$2x + 2y + 2z = 108$ 05

c. Evaluate: $(x, y) \overset{\lim}{\longrightarrow} (0,0)\,\frac{x^2 y}{x^4 + y^2}$ 04